

## Numerical techniques for design and modeling of distribution transformers

M. Tsili<sup>a</sup>, A. Kladas<sup>a,\*</sup>, P. Georgilakis<sup>b</sup>, A. Souflaris<sup>b</sup>, D. Paparigas<sup>b</sup>

<sup>a</sup> Faculty of Electrical and Computer Engineering, Laboratory of Electrical Machines, Electric Power Division, National Technical University of Athens, 9 Iroon Polytechniou Street, 15780 Athens, Greece

<sup>b</sup> Schneider Electric A.E., Elvim Plant, 32011 Inofyta Viotia, Greece

### Abstract

Power transformer analysis and design focusing on the equivalent circuit parameter evaluation by magnetic field numerical calculation is presented. The proposed method adopts a particular reduced scalar potential formulation enabling 3D magnetostatic problem solution. This method, necessitating no source field calculation, in conjunction with a mixed finite element – boundary element technique, results in a very efficient 3D numerical model for power transformer design office use. Computed results are validated through measurements. Such a methodology is very promising for investigation concerning losses and short circuit voltage variations with the main geometrical parameters. © 2004 Elsevier B.V. All rights reserved.

**Keywords:** Distribution transformers; Methodology; Geometrical parameters

### 1. Introduction

Transformers are electric machines that enable the transmission and distribution of electric energy in a simple and cost-effective way, since their efficiency overcomes 95%. The modern industry requirements necessitate the construction of a great variety of transformers that do not fit into standardized large-scale constructions. In such cases, experiential ways of electric characteristics calculation do not afford satisfying accuracy, as they concern particular geometries. Moreover, the limited delivery time does not allow the experimental verification of the predicted transformer characteristics.

Numerical modeling techniques are now-a-days well established for power transformer analysis and enable representation of all important features of these devices [1,2]. More particularly, techniques based on finite elements present interesting advantages for non-linear characteristics simula-

tion. The leakage inductance evaluation has been extensively analyzed, as well as eddy current loss in transformer tank walls, iron lamination characteristics and design considerations. The systematic increase of computer efficiency along with the evolution of numerical methods of magnetic field simulation enable the detailed transformer magnetic field analysis with the use of low cost and widely popular computational systems. The finite element method is one of the numerical methods that have prevailed in the field analysis of three-dimensional configurations that comprise materials with non-linear characteristics, like transformers, and may be applied within reasonable time in an appropriate personal computer [9,10].

On the other hand, the boundary element method is a numerical field analysis technique that uses the integral form of magnetic field equations and discretizes only the boundaries of the considered areas (in comparison to the finite element method which discretizes the whole field). Therefore, this method is suitable for open-boundary problems as well as geometries with extensive parts of air. Moreover, the combination of boundary and finite elements is widely used for electromagnetic problems since the electromagnetic field is not only confined to the conductors but it expands over extensive

\* Corresponding author. Fax: +30 210 772 3593.

E-mail addresses: mtsili@central.ntua.gr (M. Tsili), kladasel@central.ntua.gr (A. Kladas), pavlos\_georgilakis@mail.schneider.fr (P. Georgilakis), thanassis\_souflaris@mail.schneider.fr (A. Souflaris), dimitris\_paparigas@mail.schneider.fr (D. Paparigas).

parts of air, where the use of a boundary element representation can significantly decrease the computational effort [5,6].

The application of these methods to the transformer magnetic field simulation can afford accurate equivalent circuit parameters evaluation as well as contribute to the prediction of the transformer operating characteristics.

The upgrade of the distribution networks voltage (from 15 to 20 kV) has created the need for construction of transformers suitable for both voltage levels by multiple high voltage windings connection. In these cases, a difficulty of the transformer parameters evaluation through the existing design methodology arises, which can be overcome by incorporating the magnetic field analysis techniques to the accustomed approximating methods used by manufacturers. In addition, the accurate prediction of the transformer characteristics can result to its cost reduction, since the short-circuit voltage value is critical for the choice of dimensions that ensure its durability under short-circuit conditions.

## 2. Modeling techniques

### 2.1. Finite element method

In the present paper a particular scalar potential formulation has been developed, enabling the 3D magnetostatic field analysis. According to our method the magnetic field strength  $\mathbf{H}$  is conveniently partitioned to a rotational and an irrotational part as follows [3]:

$$\mathbf{H} = \mathbf{K} - \nabla\Phi \quad (1)$$

where  $\Phi$  is a scalar potential extended all over the solution domain while  $\mathbf{K}$  is a vector quantity (fictitious field distribution), defined in a simply connected subdomain comprising

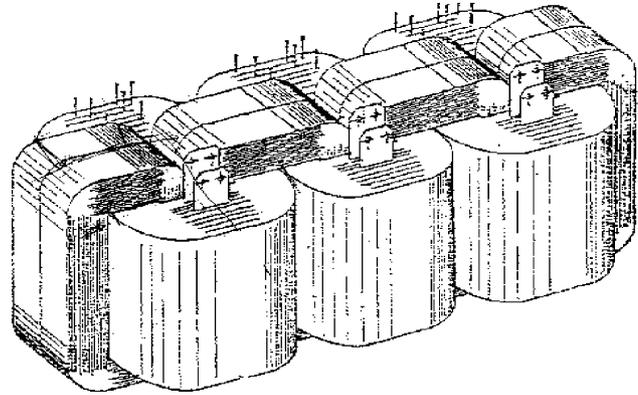


Fig. 1. Active part configuration of the three-phase shell type distribution transformer considered.

the conductor, that satisfies Ampere's law and is perpendicular on the subdomain boundary.

Fig. 1 shows active part the of the shell type distribution transformer considered. Fig. 2 illustrates the perspective view of the one-phase transformer part modeled, comprising the iron core, low and high voltage windings.

The use of this one-phase model instead of the whole three-phase transformer model was conducted for the following reasons:

- (i) The smaller model size enables the construction of more dense tetrahedral finite element mesh without great computational cost (given that the exact representation of the transformer magnetic field requires great accuracy which is dependent on the mesh density and the total execution time of the finite element calculations).
- (ii) The representation of one-phase of the active part does not affect the accuracy of the equivalent circuit parameters calculation.

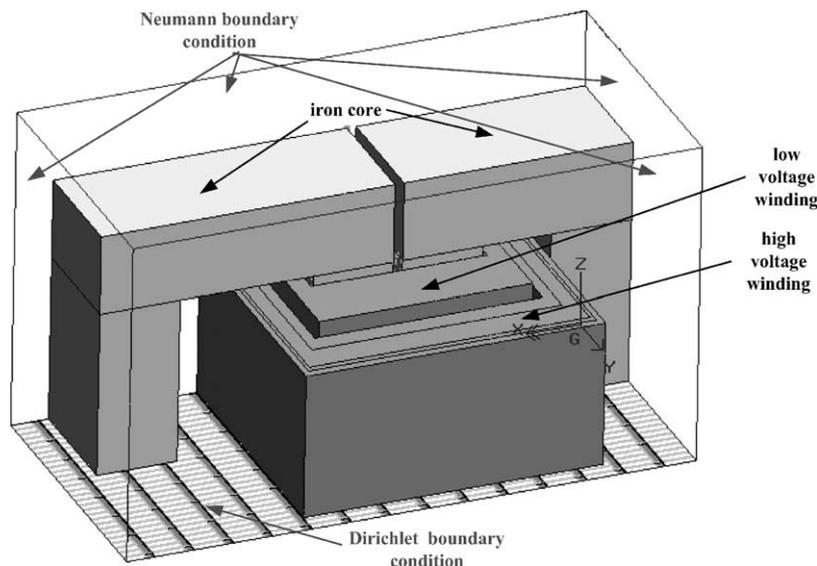


Fig. 2. Perspective view of the one-phase transformer part modeled.

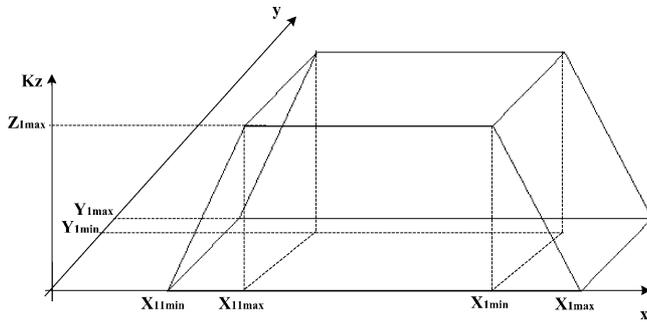


Fig. 3. Fictitious field distribution corresponding to low voltage winding.

Due to the symmetries of the problem, the solution domain was reduced to one-fourth of the device (although there is a slight disymmetry due to the terminal connections in one side). These symmetries were taken into account by the imposition of Dirichlet boundary condition ( $\Phi = 0$ ) along  $xy$ -plane and Neumann boundary condition ( $\partial\Phi/\partial n = 0$ ) along the other three faces of the air box that surrounds the transformer active part.

In such a case the distribution of  $\mathbf{K}$  is straightforward. As an example such a distribution corresponding to the low voltage winding is shown in Fig. 3. Such a formulation is well suited for finite element discretization [1,4].

As shown in Fig. 2, the high voltage winding area is divided into four subcoils. This division was made in order to take into consideration the winding connection which gives the second high voltage level (i.e. 15 kV). In particular, in the case of the first connection (20 kV) all the subcoils are considered to undergo the nominal current, while in the second one, two of them are connected in parallel, therefore, half of the nominal current flows through them. Each subcoil consists of the respective number of turns which are given by the manufacturer.

2.2. Boundary element method

The boundary element method is derived through discretization of an integral equation, that is, mathematically equivalent to the original partial differential equation. The boundary integral equation corresponding to Laplace equation is of the form:

$$c(s)\Phi(s) + \oint_{\Gamma} \left[ \Phi(s') \frac{\partial G(s', s)}{\partial n} - G(s', s) \frac{\partial \Phi(s')}{\partial n'} \right] ds' = 0 \tag{2}$$

where  $s$  is the observation point,  $s'$  the boundary  $\Gamma$  coordinate,  $n'$  the unit normal and  $G$  the fundamental solution of Laplace equation in free space.

The re-formulation of the PDE that underlies the BEM consists of an integral equation that is defined on the boundary of the domain and an integral that relates the boundary solution to the solution at the points in the domain. Therefore, while in the finite element method an entire domain mesh is

required, in the BEM formulation a mesh of the boundary only is required, resulting to the significant reduction of the problem size.

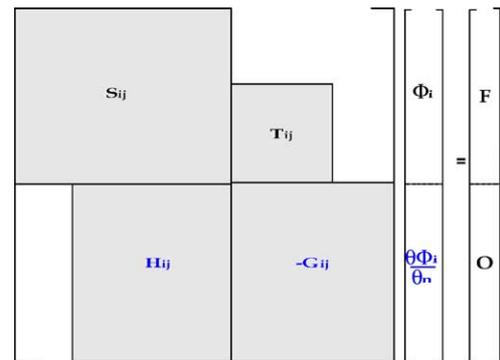
As the solution domain, in case of transformer simulation, comprises extensive parts of air, the adoption of a boundary element technique to represent the respective subdomains, improves the model performance.

2.3. Mixed FEM–BEM method

Let us consider a coupled finite element/boundary element solution domain, comprising  $m$  FE nodes,  $n$  BE nodes and  $r$  common nodes in the interface boundary. The matrices for the BE region can be written as follows:

$$\mathbf{HU} = \mathbf{GQ} \tag{3}$$

where  $\mathbf{U}$  is a vector of the BE nodal potential values,  $\mathbf{Q}$  a vector of the BE nodal  $\partial\Phi(s')/\partial n$  values, while  $H_{ij}$  and  $G_{ij}$  correspond to the Eq. (2) integrals  $\oint_{\Gamma_j} (\partial G_i(s', s)/\partial n) ds'$  and  $\oint_{\Gamma_j} G_i(s', s) ds'$ , respectively [6]. The matrices of the FE region can be written as  $\mathbf{SU} = \mathbf{F}$ , where  $\mathbf{S}$  is the stiffness matrix and  $\mathbf{F}$  the source vector. Therefore, the global system matrix has the form



where  $T_{ij}$  is the term used to link the finite element region to the boundary element region (involving the potential and normal derivative values of the FE–BE interface boundary nodes), [7,8].

3. Results and discussion

The proposed reduced scalar potential formulation has been applied in the 3D numerical analysis of a transformer under short circuit for its leakage reactance calculation. Two study cases were considered, and the respective results of the finite element analysis were compared to the experimental results (local field values and short-circuit voltage value).

3.1. Study Case 1

The case of the one-phase part of a 1000 kVA, dual voltage 20–15 kV/400 V three-phase shell type power transformer, has been considered.

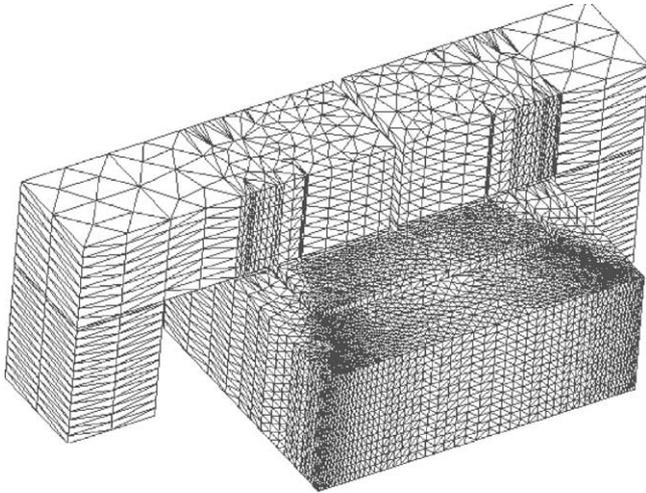


Fig. 4. Three-dimensional finite element mesh of the transformer active part (89,603 nodes).

Fig. 4 illustrates the finite element tetrahedral mesh adopted for the calculation of the transformer magnetic field under short-circuit conditions. The mesh consists of 89,603 nodes, a density which is considered satisfactory (although not optimum) for a three-dimensional problem. As shown in this figure, it is significantly more dense in the windings area, while it is sparser in the iron cores. This configuration was chosen in order to represent in great detail the magnetic field sources (i.e. the windings) without extreme increase of the total number of nodes (by reducing the respective number of nodes in the cores area).

Fig. 5 shows the magnetic flux density magnitude distribution during short-circuit test, as it was calculated by the proposed 3D finite element method with the use of the above mesh.

These field values have been compared to those measured by a Hall effect probe during short-circuit test. Fig. 6 gives

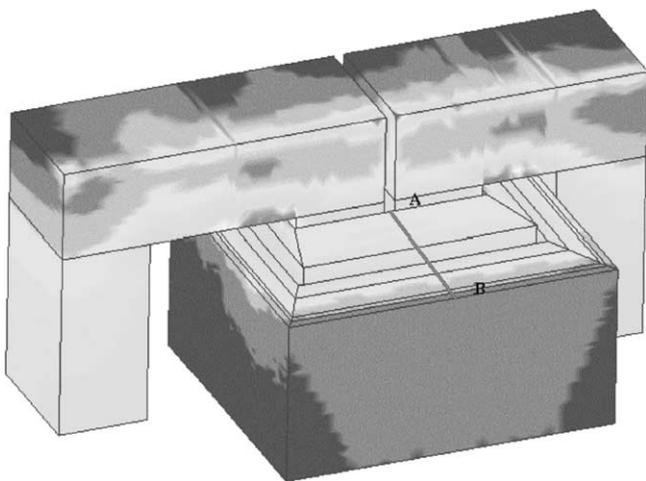


Fig. 5. Magnetic flux density magnitude distribution during short-circuit test.

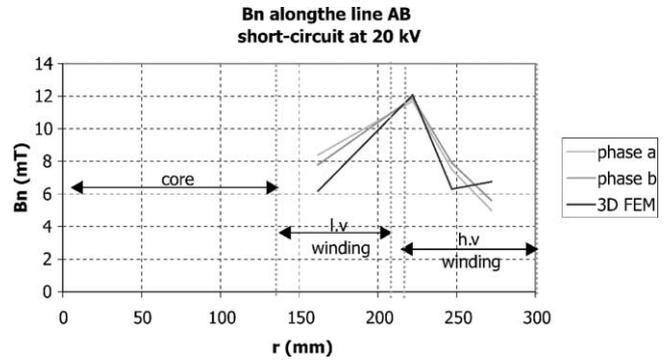


Fig. 6. Comparison of measured and computed field values along the line AB.

the variation of the perpendicular flux density component  $B_n$  along the line AB, positioned as shown in Fig. 5, in case of short-circuit with the high voltage winding connections corresponding to 20 kV voltage supply. This figure illustrates the good correlation of the simulated results with the local leakage field measurements.

Table 1 compares the measured short-circuit voltage value deduced by the short-circuit test and the calculated one with the use of different mesh densities, in order to evaluate the respective variation of the error. The error of the short-circuit voltage calculation reduces significantly as the mesh density increases, while it appears to be similar for the two high voltage levels.

### 3.2. Study Case 2

The case of the one-phase part of a 630 kVA, dual voltage 20–15 kV/400 V three-phase shell type power transformer, was also considered. The three-dimensional finite element model that was used is similar to the one used for the first study case, consisting of a 100,999 nodes tetrahedral mesh.

The computed values of the perpendicular flux density component  $B_n$  have been compared to those measured by a Hall effect probe during short-circuit test along the lines AB and CD of Fig. 7. Figs. 8 and 9 show the variation of  $B_n$  along the line AB with the high voltage winding connections corresponding to 20 and 15 kV voltage supply, respectively, while Figs. 10 and 11 give the variation of  $B_n$  along the line CD. In

Table 1  
Comparison of calculated and measured short-circuit voltage values for the 1000 kVA transformer modeled

Number of nodes	Calculated short-circuit voltage	Measured short-circuit voltage	Error (%)	High voltage level (kV)
14491	6.38	6.13	4.00	20
49047	6.30		2.84	
86903	6.19		0.98	
14491	6.13	5.95	3.01	15
49047	6.09		2.30	
86903	5.97		0.34	

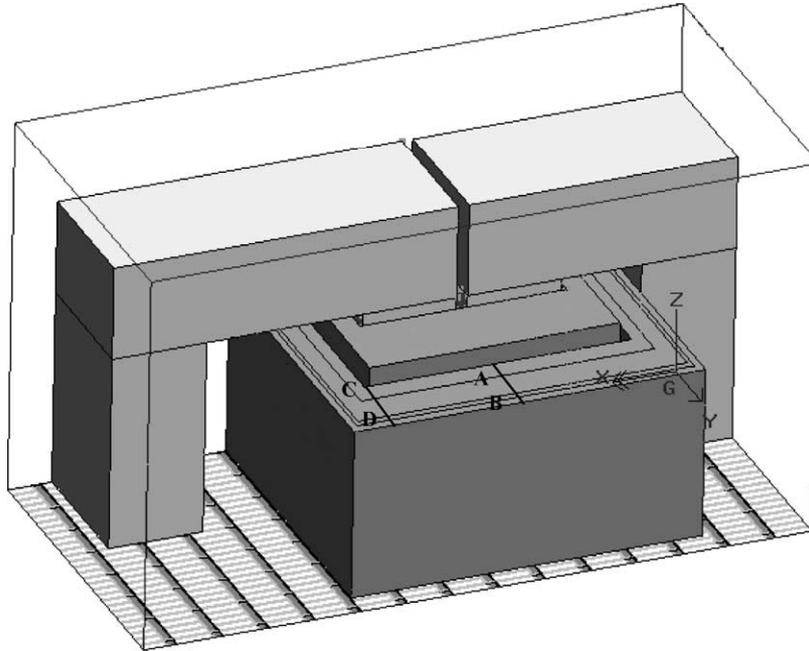


Fig. 7. One-phase 3D model of the 630 kVA distribution transformer.

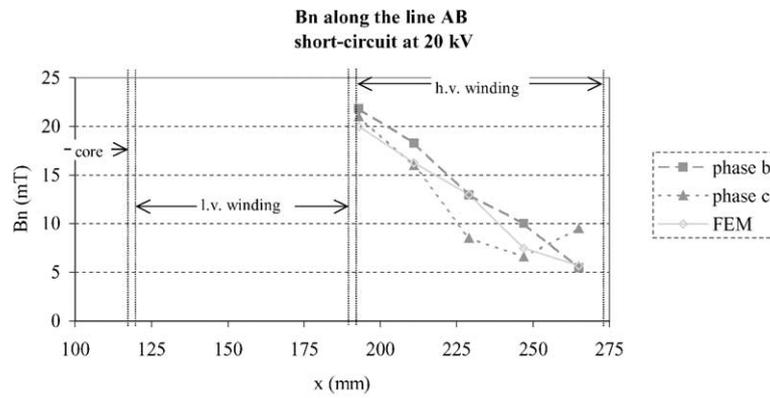


Fig. 8. Comparison of measured and computed field values along the line AB (short-circuit at 20 kV).

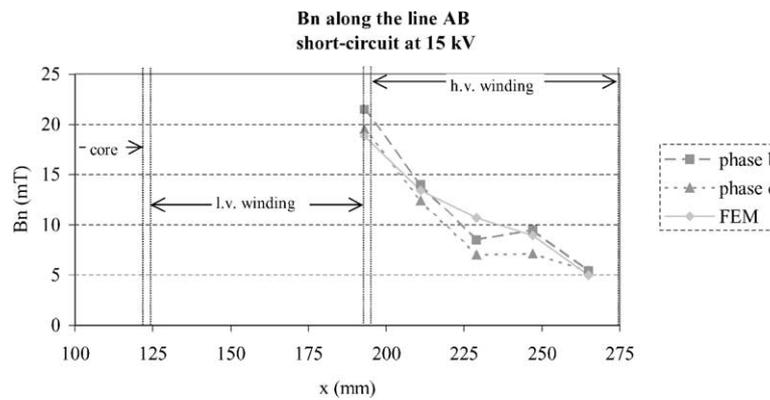


Fig. 9. Comparison of measured and computed field values along the line AB (short-circuit at 15 kV).

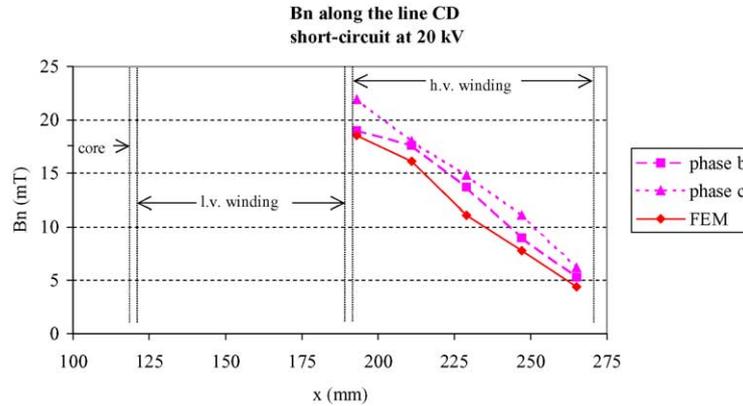


Fig. 10. Comparison of measured and computed field values along the line CD (short-circuit at 20 kV).

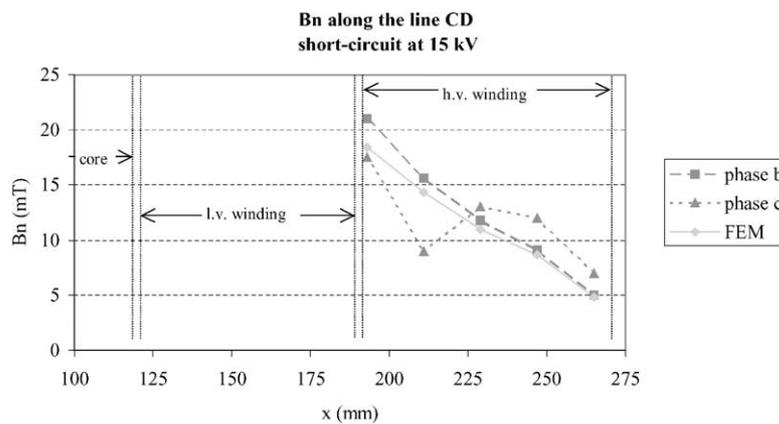


Fig. 11. Comparison of measured and computed field values along the line CD (short-circuit at 15 kV).

the case of AB line, the computed and measured field values at 20 kV are very close, with the exception of two points in the “phase c” curve (due to errors during the measuring process). The proximity is also good for the case of 15 kV, where we can observe the field variation due to the different current level going through the two high voltage subcoils connected in parallel. The same conclusions are drawn from the observation of the curves corresponding to the line CD for the case of short-circuit at 20 and 15 kV.

Table 2 compares the measured short-circuit voltage value deduced by the short-circuit test and the calculated one with

Table 2  
Comparison of calculated and measured short-circuit voltage values for the 630 kVA transformer modeled

Number of nodes	Calculated short-circuit voltage	Measured short-circuit voltage	Error (%)	High voltage level (kV)
23696	5.95	5.61	5.97	20
47044	5.73		2.07	
100999	5.67		1.14	
23696	5.75	5.50	4.55	15
47044	5.62		2.13	
100999	5.57		1.18	

the use of different mesh densities, in order to evaluate the respective variation of the error. The variation of the error is similar to that of Table 1, although in the case of the 630 kVA transformer the error is relatively greater for medium mesh densities (>4% for a 23,696 nodes mesh) and reaches a satisfactory low value for a mesh of 100,000 nodes.

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**M. Tsili** was born in Greece, in 1976. She received the Diploma in Electrical and Computer Engineering from the National Technical University of Athens in 2001 where she follows post-graduate studies. Her research interests include transformer and electric machine modeling as well as analysis of generating units by renewable energy sources.

**A. Kladas** was born in Greece, in 1959. He received the Diploma in Electrical Engineering from the Aristotle University of Thessaloniki, Greece in 1982 and the DEA and PhD degrees in 1983 and 1987, respectively, from the University of Pierre and Marie Curie (Paris 6), France. He served as Associate Assistant in the University of Pierre and Marie Curie from 1984 to 1989. During the period 1991 to 1996 he joined the Public Power Corporation of Greece, where he was engaged in the System Studies Department. Since 1996 he joined the Department of Electrical and Computer Engineering of the National Technical University of Athens (NTUA), where he is now Associate Professor. His research interests include transformer and electric machine modeling and design as well as analysis of generating units by renewable energy sources and industrial drives.

**P. Georgilakis** (S'1998, M'2001) was born in Chania, Greece in 1967. He received the Diploma in Electrical and Computer Engineering and the PhD degree from the National Technical University of Athens, Greece in 1990 and 2000, respectively. In 1994 he joined Schneider Electric AE, Greece. He has worked in the Development and also the Quality Control Departments of the Industrial Division of the company. From 1999 to 2001 he was the R&D Manager of Schneider Electric AE. At present he is Business and Activity Manager in the Marketing Division. He is member of IEEE, CIGRE, and the Technical Chamber of Greece.

**A. Souflaris** was born in Athens, Greece in 1956. He received the Diploma in Electrical Engineering from the Technical University of Pireaus, Greece in 1981. He joined Schneider Electric AE in 1985 as Transformer Design Engineer and from 1988 he is the Transformer Design Manager of Schneider Electric AE.

**D. Paparigas** was born in Komotini, Greece in 1945. He received the Diploma in Electric Machines and Instruments from the Moscow Energy Institute, USSR in 1972. From 1974 to 1976 he worked in the Engineering Department of Masina Xrisoulouris, Greece. Since 1976, he has been with Schneider Electric AE (former Elvim), Greece. He has been engaged in several positions, such as Quality Control Manager, Production Manager, Transformer Design Manager and Quality Assurance Director. At present, he is Technical Division Director and Industrial Division Director.